Maths for Computing
Assignment 4

1. *(5 marks)* Let $n \geq 1$. Prove that if we select any $n+1$ integers from $[2n]$, then there exists two integers, say a and b, such that $a \,\%\, b = 0$.

2. *(3 marks)* How many ways are there to select an 11-member soccer team and a 5-member basketball team from a class of 30 students if
a) nobody can be on two teams.
b) any number of students can be on both the teams.
c) at most one student can be on both the teams.

3. *(5 marks)* Give a combinatorial proof of the following equation, i.e., prove that both sides are counting the same thing, where $1 \leq k \leq n$ and k, n are integers. (Modifications on any side is not allowed)

$$1.\binom{n-1}{k-1} + 2.\binom{n-2}{k-1} + \ldots + (n-k+1).\binom{k-1}{k-1} = \binom{n+1}{k+1}$$

4. *(5 marks)* Suppose $X = [12]$ and $Y = [8]$. How many functions f from X to Y satisfy the property that $|f(X)| = 5$? How many satisfy the property that $|f(X)| \leq 5$?

5. *(5 marks)* Prove that $p_3(n) = |X|$, where X is the set of 3 length partitions of $2n$, where every element of the partition is at most $n-1$.

6. (5 marks) Prove that if $n \geq 2$, then $n! < S(2n, n) < (2n)!$.